

## EXPERIMENT 9: COMPTON EFFECT

### Related Topics

Interactions of photons with electrons, conservation of momentum and energy, inelastic and elastic scattering, interaction cross section, Compton wavelength.

### Principle

When photons are scattered on electrons, their momentum and energy gets changed. The change of energy in dependence on scattering angle is measured by g-spectroscopy. The Compton wavelength is determined from the measurement data.

### Tasks

- Calibrate the set-up with the aid of the 34 kBq  $^{137}\text{Cs}$  and the 22Na (74kBq) source.
- Measure spectra of the radiation scattered by a steel rod at various angles.
- Calculate the Compton wavelength from the spectra.

### Theory and Evaluation

Compton scattering is elastic scattering of photons on electrons. Both momentum and energy conservation are fulfilled and momentum and energy are exchanged between photon and electron the same way as one would expect of mass points. The energy range is that of the electron's resting mass so the calculation has to be relativistic.

The energy  $E$  of a photon depends on its frequency  $f$  and the Planck constant  $h$  and is

$$E = hf$$

and the momentum  $p$  of a photon with wavenumber  $k$  is

$$p = \hbar k = \frac{hf}{c}$$

If the electron is at rest at the beginning, all the momentum comes from the photon and stays constant throughout the reaction. After the reaction the initial momentum is split up into the momentum of the photon. The photon moves now under the angle  $\varphi$  to its initial direction and has the altered frequency  $f'$ . The momentum of the electron after the scattering is  $p_e = mv$  with the mass as a function of velocity ( $m \geq m_e$ ). The vector sum of both equals the initial momentum and thus holds for the squares of the vectors:

$$m^2 v^2 = \frac{h^2}{c^2} (f'^2 + f^2 - 2f' \cdot f \cdot \cos \varphi)$$

The energy has to be constant throughout the reaction, too and for simplicity counting the electron resting mass equivalent in the equation (the point of zero energy is arbitrary)

$$hf + m_e c^2 = hf' + mc^2$$

with the relativistic relation for electron speed  $v$

$$m_e^2 = m^2 \left( 1 - \left( \frac{v}{c} \right)^2 \right)$$

$$m^2 v^2 c^2 = m^2 c^4 - m_e^2 c^4$$

$$m^2 c^4 - m_e^2 c^4 = h^2 (f'^2 + f^2 - 2f' \cdot f \cdot \cos \varphi)$$

$$m^2 c^4 - m_e^2 c^4 = h^2 (f - f')^2 + 2m_e m c^4 - 2m_e c^4$$

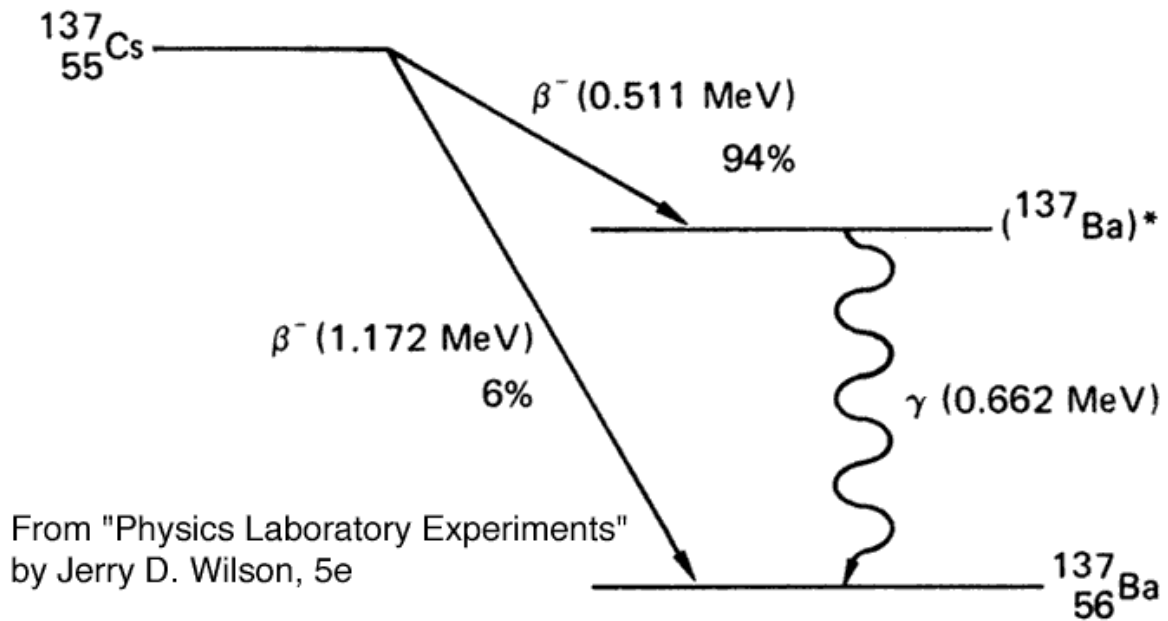
$$h^2 \cdot f \cdot f' (1 - \cos \varphi) = m_e (m - m_e) c^4 = m_e c^2 \cdot E_{\text{kin}}$$

$$h^2 \cdot f \cdot f' (1 - \cos \varphi) = m_e c^2 h (f - f')$$

$$f = \frac{c}{\lambda}, f' = \frac{c}{\lambda'} \text{ and } \Delta\lambda = \lambda' - \lambda = \frac{c(f - f')}{f f'} \text{ is}$$

$$\Delta\lambda = \frac{h}{m_e c} (1 - \cos \varphi)$$

which is Compton's formula for the wavelength shift.



**Fig.1:** Decay scheme of Cs-137. Most of the cesium-137 (Cs-137) nuclei (94%) decay to an excited state of barium-137 ( $^{137}\text{Ba}^*$ ), which then gamma decays to a stable state.

$$\frac{h}{m_e c} = \lambda_0$$

is called Compton wavelength, with

$$E = hf = \frac{hc}{\lambda}$$

is the corresponding energy  $E_c = m_e c^2$  the energy equivalent of the electron's resting mass of 511 keV. So the energy scale for the Compton effect is set by the electron mass which can be interpreted in the way that an elastic scattering occurs when the photon is about as "heavy" as the electron.

Fig. 1 shows the decay scheme of the used nuclid  $^{137}\text{Cs}$ . The proportions of the energy scale are not displayed correctly. The main emission is the sharp 662 keV line. But there is a probability of the  $11/2^-$  excited state of the daughter nuclid  $^{137m}\text{Ba}$  to decay by inner conversion because of the high difference of spin quantum number between ground and excited state – in the case of  $^{137m}\text{Ba}$  leading to a 32 keV X-ray line and a long half life of the excited state of two and a half minute. Only the photons reach the detector, electrons cannot leave the source pellet.

Incoming photons of a defined energy are detected in the scintillation counter by the electrons they excite in the scintillator crystal either by photo effect, where they transfer all their momentum and energy to the electron (photon absorption or completely inelastic scattering), or by Compton scattering, where momentum and energy are exchanged between photon and electron in an elastic scattering. So the measurement result of a sharp photon energy is a whole spectrum consisting of the photo peak and the Compton background. The Compton scattering the photons undergo in the detector is here not of interest – only the incidents detected by photo effect thus showing the original photon energy should be counted for as interesting incidents leaving other effects in the spectrum ignored – you are searching for the photo peak of photons Compton scattered by the steel rod. The set-up is made in a way that only those of the 662 keV photons that have undergone scattering on the steel rod in a defined angle reach the detector and can be counted and their energy is evaluated in the recorded g-spectrum. The dependence of the energy  $E'$  of the scattered photons that had reached the steel rod with energy  $E_0$  (here 662 keV) on the scattering angle is then:

$$E' = \frac{1}{\frac{1}{m_e c^2}(1 - \cos \varphi) + \frac{1}{E_0}}$$

This function has a maximum for  $\varphi = 180^\circ$ , where no energy is transferred to the electron and the photon keeps its initial energy and a minimum for  $\varphi = 0^\circ$ , where the fraction of the initial energy

$$\frac{E'}{E_0} = \frac{1}{\frac{E_0}{m_e c^2} + 1}$$

is reflected.

For the spectrum of the electrons exited in the steel rod by Compton effect, though not measured, the following can be said: In the case of Compton scattering there is a highest energy fraction of the initial energy of the photon that can be transferred to the electron. This energy is transferred, if the photon gets reflected back into the direction where it came from (back scattering).

### **Equipment**

Cs-137 37 kBq, Cs-137 18.5 MBq, Na-22 74 kBq sources, lead blocks, gamma detector

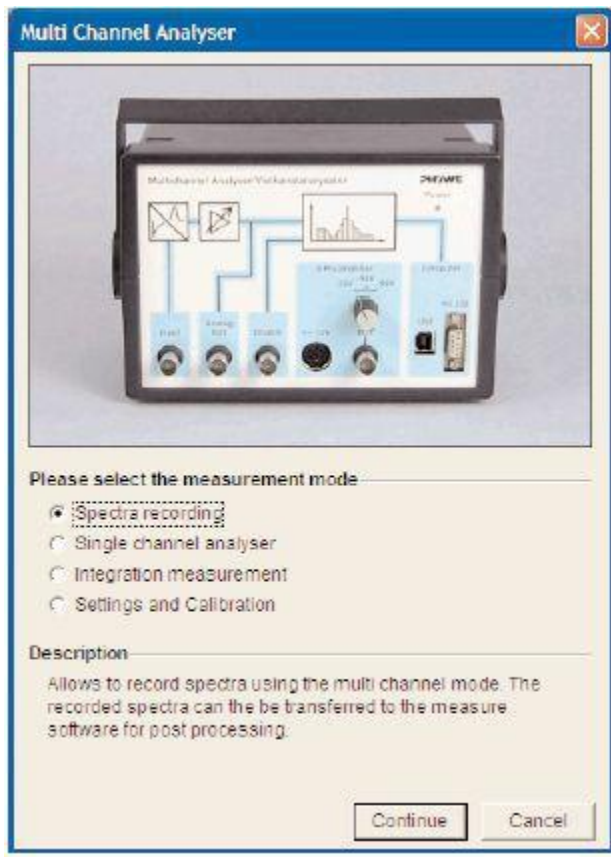


**Fig.2:** Experimental set-up for Compton effect.

### **Set-up and Procedure**

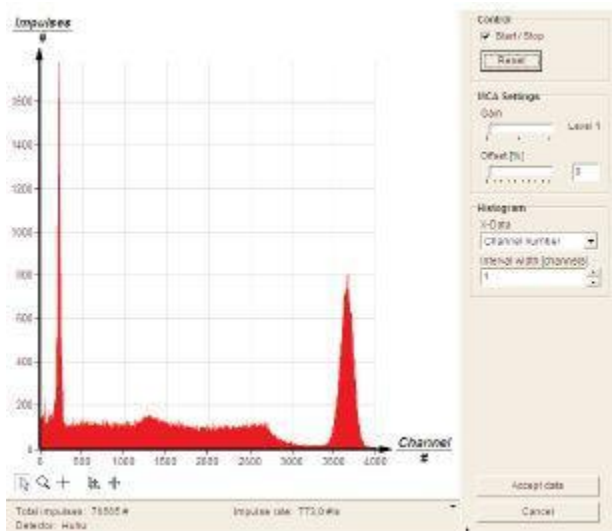
Set up the equipment as seen in Fig. 2. Before turning on the operating unit for the scintillation counter, connect the high voltage cable correctly to operating unit and photomultiplier and read the instructions in the manual of the gamma-detector. Set the multiturn potentiometer of the operating unit to 2.00. Connect the multi channel analyzer (MCA) to the computer's USB port or a RS232 serial interface and start the "measure" program. Select the Gauge "Multi Channel Analyzer".

1. Select "Spectra recording" (see Fig. 3), set the "Gain" to "Level 1", set the "Offset" to 0 %, choose "Channel number" as x-Data (see Fig. 4) and start data recording. Put the 137-Cs pellet into the slot of the screening cylinder for the g-detector near to the detector. Increase the high voltage with the multiturn potentiometer until the 662 keV peak gets to about channel 3500. If you are not sure about the high voltage setting, start with low settings and increase the high voltage, until the 662 keV peak wanders to the right end of the spectrum. Cancel the measurement and leave the high voltage setting unchanged from now on.

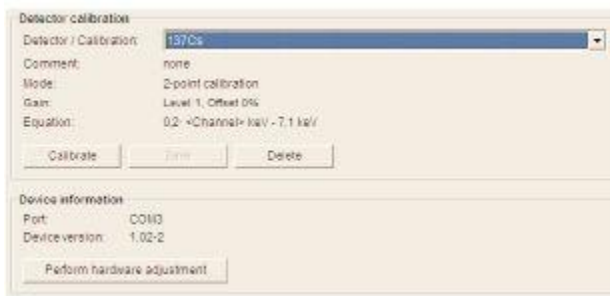


**Fig. 3:** Start window of the MCA

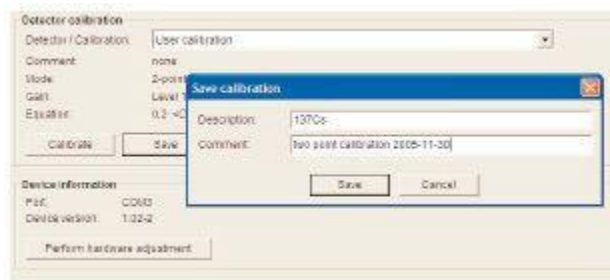
Now start the MCA gauge again and select "Settings and calibration". The window seen in Fig. 5 appears. Use the "Calibrate" button. Perform the 3-point calibration with the 22-Na and the 137-Cs source and use the "Apply" button. Save the calibration as seen on Fig. 6. Remove the 34 kBq source from the slit of the cylindrical detector screening. Be careful with detector handling – the scintillation crystal breaks easily and cracks in it will reduce sensitivity and energy resolution.



**Fig. 4:** Window for spectrum recording – here the spectrum of 137-Cs with gain level 1.



**Fig. 5:** Calibration window



**Fig. 6:** Saving the calibration

**2.** Now return to "Spectra recording" and set the "x-data" to "Energy". Put the 18.5 MBq source into the lead brick with hole. Adjust the setup in a way that mainly scattered photons from the steel rod get recorded. The 662 keV line should be well suppressed by the shielding. The presence of the steel rod should increase the impulse rate distinctly – if you take it away, it should drop clearly. The spectrum should show a distinct maximum from the preferred Compton scattering angle. Record such a spectrum for each angle (e.g. for 30°, 60°, 90°, 120°) and save the data with the "Accept data" button. Start with high or medium angles.

**3.** Save the recorded spectra to your disc drive. Different ways may be suitable for finding the maximum of the Compton scattering peak: You may use "Analysis" > "Smooth..." to reduce the statistical noise in the spectrum and evaluate the peak position with the "Survey" tool. Else you may mark the interesting region of the spectrum with the "Mark" tool that looks like a cross and then use "Analysis" > "Function fitting..." to fit a scaled normal distribution with offset to the data and then use "Analysis" > "Show extrema..." to find the position of the peak of the fit. Plot the Compton scattering peak energy versus the scattering angle. Plot the wavelength  $\lambda = hc/E$  that corresponds to the energy E over the scattering angle and the wavelength shift, too.