Motion on an Inclined Plane

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1 Theory

Average acceleration is the change of a particle's velocity over a specific period of time. This physical quantity can be defined mathematically as,

$$\vec{a}_{av} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1}, \tag{1}$$

where $\vec{V}_1$ and $\vec{V}_2$ is the velocity at t_1 and t_2, respectively. And **Instantaneous acceleration** is the acceleration of a particle at a specific moment of time. Instantaneous acceleration can be approached by minimizing the time interval in Eq. 1 such as,

$$\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1} \equiv \frac{d\vec{V}}{dt}, \tag{2}$$

where Δt is the difference between t_2 and t_1.

1.1 Newton's 2nd Law of Motion

The second law states that the change of momentum of a particle is directly proportional to the force applied on it,

$$\vec{F}_{eq} = \frac{d\vec{P}}{dt}. \tag{3}$$

Since the momentum of a particle is multiplication of its mass and velocity, this law can be defined via a particle's acceleration, too.

$$\vec{F}_{eq} = \frac{d}{dt}(m\vec{V}) = m\frac{d\vec{V}}{dt} \qquad \Rightarrow \qquad \vec{F}_{ins} = m\vec{a}. \tag{4}$$

This last version of the law is only valid for particles or systems whose mass does not change over time.

1.2 Motion on an Inclined Plane

Consider a particle moving down on a frictionless inclined plane shown in Fig 1. Due to the influence of gravity, the velocity of particle changes over time. Therefore, this motion is a motion with acceleration. Net force on the puck is the gravitational force and normal force of the surface.

$$\vec{F}_{eq} = \vec{F}_{gravity} + \vec{N} = m\vec{g} + \vec{N}. \tag{5}$$

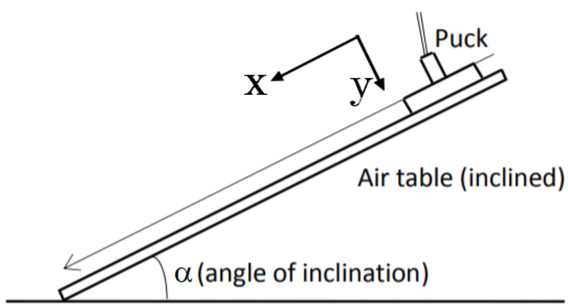

Figure 1: Path of a thrown object.

On the other hand, according to Newton's second law of motion, net force is,

$$\vec{F}_{eq} = m\vec{a}. \tag{6}$$

By using Eq. 5 and Eq. 6 together, one can get the equation of motion for puck on an inclined plane.

$$m\vec{g} + \vec{N} = m\vec{a}. \tag{7}$$

At this point, we need to choose a coordinate system and we choose the coordinates as shown in Fig. 1. Now we rewrite Eq. 7 in components of this coordinate system, such as,

$$mgsin(\alpha)\hat{i} + mgcos(\alpha)\hat{j} + N(-\hat{j}) = ma_x\hat{i} + ma_y\hat{j} \tag{8}$$

Since x and y coordinates are linearly independent, we may decompose this equation as follows,

$$\hat{i} \quad : \quad mgsin(\alpha) = ma_x, \tag{9}$$
$$\hat{j} \quad : \quad mgcos(\alpha) - N = ma_y. \tag{10}$$

Because there is no motion in y direction, net force and acceleration in that direction is zero.

$$mgcos(\alpha) = N, \quad a_y = 0. \tag{11}$$

On the other hand, by following Eq. 9, we obtain the acceleration of the puck.

$$a_x = gsin(\alpha). \tag{12}$$

2 Procedure

2.1 Experiment Procedure

1. Turn on the lab table and then the airtable.
2. By using only compressor's pedal, make sure the airtable is working.
3. If the airtable is working; turn it off. If not; contact with your lab instructor.
4. By using the cylindrical piece on yout experiment table, turn the airtable into an inclined plane.
5. Place the carbon paper into the airtable and experiment sheet onto the carbon paper.
6. Set the frequency of the airtable to 20 Hz and write this value down in the Table in Section 3.
7. Turn on the experiment table and the airtable.
8. Place one of the pucks at the upper part of the airtable. Secure the other one by a piece of folded paper at lower part of the table.
9. Push both of the pedals and let the puck move down. Be careful about that once the puch reaches the bottom, it will be reflected by the bottom edge of the airtable. You should remove your hand from the pedals just before the puck is reflected.
10. Turn the airtable and the experiment table off.
11. Move the experiment sheet from the airtable to the experiment table.

2.2 Analysis Procedure

1. Determine the very first dot left by the puck and label it as 0.
2. Label the other dots in an increasing manner shown in Fig. 2.
3. With a ruler, measure the length of interval between 0th and n_th dots on the experiment paper. Write the numerical values down on Table(1) in Section(3).
4. Plot the x-t^2 graph using data on Table(1).
5. Calculate the slope of the graph and write it down on Section(3). This yields the half of the experimental acceleration.

$$slope = \frac{1}{2} a_{exp}. \tag{13}$$

   By using the slope of the plot, calculate a_{exp}.

6. By using percentage error formula,

$$P.E. = \% \frac{|a_{the} - a_{exp}|}{a_{the}} \times 100 \tag{14}$$

   calculate the percentage error in a_{exp}.

7. By using the data on Table 1, fill the Table 2.
8. Plot $\log x$ vs. $\log t$ graph. Slope of this graph should be 2 and y-intercept of it is related to the experimental acceleration.
9. Analyze the log plots according to Graph Guide.
10. Write example calculations for Table(1) on Section(3).

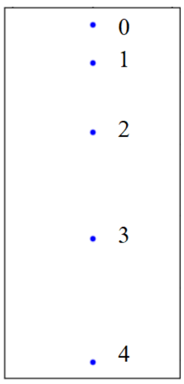

Figure 2: Experiment Sheet.

3 Data & Results

- Frequency of airtable: $f =$ ____________________.

Table 1: ____________________

#	x (cm)	t (sec)	t^2 (sec^2)
1			
2			
3			
4			
5			

Table 2: ____________________

#	x (cm)	$\log x$	t (sec)	$\log t$
1				
2				
3				
4				
5				

3.1 Example Calculations

- Calculations for t^2:

- Calculations for $\log x$:

- Calculations for $\log t$:

- Calculations for percentage error of slope of Plot x-t^2:

- Calculations for percentage error of slope of Plot $\log x$-$\log t$:

- Calculations for percentage error of y-intercept of Plot $\log x$-$\log t$:

4 Conclusions

5 Notes