Projectile Motion

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1 Theory

Projectile motion is the motion of a thrown object with an angle to the surface of Earth or from a height with 0°. So the motion occurs under the influence of only the gravitational acceleration. The force acts upon the thrown object is,

$$\vec{F}_{net} = m\vec{g}. \tag{1}$$

If the coordinate system is chosen as in Figure (1) and Newton's 2nd law of motion is considered; one gets,

$$m\vec{a} = -mg\hat{\mathbf{j}}. \tag{2}$$

Since the motion is 2 dimensional the acceleration of motion is also 2 dimensional.

$$\vec{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}. \tag{3}$$

When Eqn (2) and Eqn (3) are taken into account together, the acceleration components can be found as,

$$a_x = 0, \qquad a_y = -g. \tag{4}$$

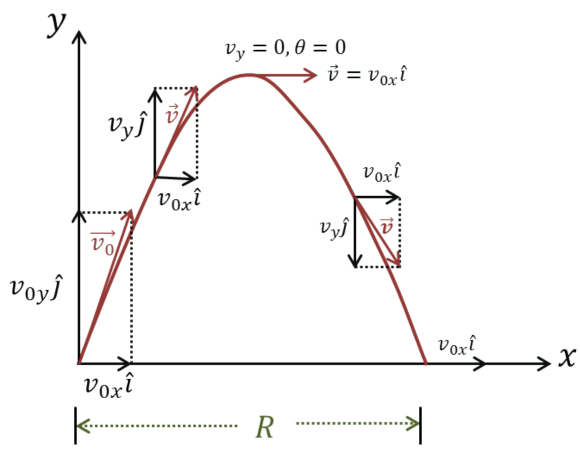

Figure 1: Path of a thrown object.

We perform our experiments on the airtable and the environment with gravitational acceleration is simulated on the inclined plane. Thus the projection of the motion onto the inclined plane needs to be reckoned.
On the inclined plane, the net force on the object is (Fig (2)),

$$\vec{F}_{net} = -mg\sin\alpha\hat{\mathbf{j}} \qquad \Rightarrow \qquad \boxed{a_x = 0, \quad a_y = -g\sin\alpha.} \tag{5}$$

Recall the general definition of acceleration;

$$a \quad = \quad \frac{dV}{dt} \qquad \Rightarrow \qquad \int \frac{dV}{dt}dt = \int adt \tag{6}$$

Then for 2 dimensions of projectile motion, we can get,

$$\underbrace{\int_0^t \frac{dV_x}{dt'}dt'}_{=V_x(t')|_{t'=0}^{t'=t}} = \underbrace{\int_0^t 0dt'}_{=0} \qquad \Rightarrow \qquad \boxed{V_x(t) = V_x(0) = V_{0x}} \tag{7}$$

$$\underbrace{\int_0^t \frac{dV_y}{dt'}dt'}_{=V_y(t')|_{t'=0}^{t'=t}} = \underbrace{\int_0^t (-g\sin\alpha)dt'}_{=-g\sin(\alpha)t'|_{t'=0}^{t'=t}} \qquad \Rightarrow \qquad \boxed{V_y(t) = V_{0y} - g\sin(\alpha)t} \tag{8}$$

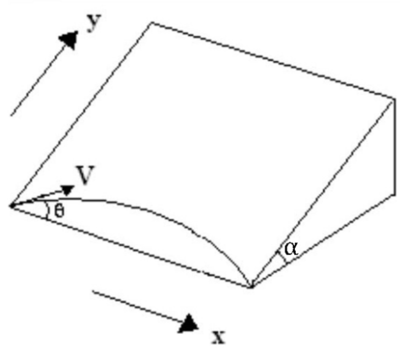

Figure 2: Schematic representation for projectile motion on an inclined plane[1].

Taking a second integrate of acceleration with respect to time yields the location formulas for projectile motion on an inclined plane as follows,

$$x(t) = x_0 + V_{0x}t, \tag{9}$$
$$y(t) = y_0 + V_{0y}t - \frac{1}{2}g\sin(\alpha)t^2. \tag{10}$$

According to Eqn (9) the motion in x-direction is motion with constant velocity and to Eqn (10) the motion in y-direction is motion with constant acceleration. The initial velocity,

$$V_0 = V_{0x}\hat{\mathbf{i}} + V_{0y}\hat{\mathbf{j}}, \tag{11}$$

where $V_{0x} = V_0\sin(\theta)$ and $V_{0y} = V_0\cos(\theta)$ dies away over time until a certain point due to the negative acceleration in y-direction meanwhile the height of the object increases. This certain height is the maximum height value of the object, namely the h_{max}. At h_{max} the y-component of the velocity vanishes momentarily and it begins to gain value in opposite direction immediately. Simultaneously, after h_{max} the height starts to decrease.
Remember, there is only acceleration in y-direction. This means that the x-component of the velocity does not change at all and the distance taken in x-direction per unit time remains same. This is the literal definition of motion with constant velocity. The time elapsed during the motion is called as "*the time of flight*", t_{fli}. If the object is thrown from the ground, then the total time until it reaches to h_{max} is half of t_{fli}. Let's write this time value in Eqn (8) to derive a formula for it.

$$V_y\left(\frac{t_{fli}}{2}\right) = V_{0y} - g\sin(\alpha)\frac{t_{fli}}{2}. \tag{12}$$

Since we know that the y-component of velocity vanished at h_{max}, then the left-hand-side (LHS) of Eqn (12) also vanishes and results in,

$$t_{fli} = \frac{2V_{0y}}{g\sin(\alpha)} = \frac{2V_0\sin(\theta)}{g\sin(\alpha)}. \tag{13}$$

Recall that the h_{max} is the maximum height of the object can reach from its initial y-location and the object reaches to h_{max} at $t = t_{fli}/2$. Then the definition for h_{max} is,

$$h_{max} \equiv y\left(\frac{t_{fli}}{2}\right) - y_0. \tag{14}$$

One can write Eqn (13) into Eqn (10), to obtain h_{max} in terms of initial values.

$$h_{max} = \frac{V_0^2\sin^2(\theta)}{2g\sin(\alpha)} \tag{15}$$

Lastly, the maximum distance gathered in x-direction is called as "*the range*",

$$R \equiv x(t_{fli}) - x_0. \tag{16}$$

By using Eqn (9), we get,

$$R = \frac{2V_0^2\cos(\theta)\sin(\theta)}{g\sin(\alpha)}, \tag{17}$$

or,

$$R = \frac{V_0^2\sin(2\theta)}{g\sin(\alpha)}. \tag{18}$$

2 Procedure

2.1 Experiment Procedure

This experiment will be performed several times with different θ values with the exact same procedure. First three θ values are given in Table(1). The last 2 columns in Table(1) is reserved for the additional θ values which are going to be determined by the student. You should choose 2 complimentary θ angles. Before you concretize your choices, make sure they are adjustable on the handgun.

1. Turn the airtable into an inclined plane by placing its cylindirical part under its back foot.
2. Turn on the lab table and then the airtable.
3. By using only compressor's pedal, make sure the airtable is working.
4. If the airtable is working; turn it off. If not; contact with your lab instructor.
5. Place the carbon paper into the airtable and experiment sheet onto the carbon paper.
6. Set the frequency of the airtable to $40Hz$ or $50Hz$ and write this value down in Section 3.
7. Fasten the handgun to the airtable as shown in the Figure ().
8. Set the angle of the handgun to the 1st θ value in Table (1).
9. Turn on the experiment table and the airtable.
10. Pull puck against the rubber band of the handgun and hold it as is to set the handgun.
11. Push both of the pedals and let the puck go. It should make a projectile motion.
12. Turn the airtable and the experiment table off.
13. Move the experiment sheet from the airtable to the experiment table and write the θ value on it.
14. Repeat steps 8-13 with next θ value from Table (1) with a new experiment sheet.

2.2 Analysis Procedure

Consider the experiment sheets one by one and analyse them as follows.

1. Determine the very first dot left by the puck and label it as 0.
2. With a ruler, measure the length of interval between 0th and 1st dots, l_0, on the experiment paper. Using this value and the frequency of the airtable determine the velocity. Write the numerical values down on Section(3).
3. By using θ, calculate and record the x- and y-components of the initial velocity.
4. Calculate the theoretical values for t_{fli}, h_{max} and R and fill the table. Write your example calculations for percentage error in details.
5. Using a ruler, draw a horizontal line (level line) originated from the 0th dot.
6. Determine the last dot of the projectile motion which coincides the level line. (You should ignore the dots below the level line.)
7. Measure all of the experimental values for h_{max} and R with a ruler and note them on Table(1). Treat the level lines as the ground level.
8. Count the number of intervals created during the motion and calculate the flight time and fill the related blanks on Table(1).
9. Calculate the percentage errors. Write your example calculations for percentage error in details.

3 Data & Results

- Frequency of airtable: $f =$ ________________.

Table 1: Data taken in the experiment.

$\theta \pm \Delta\theta$		$45 \pm$ ___	$30 \pm$ ___	$60 \pm$ ___		
$l_0 \pm \Delta l_0 (cm)$						
Initial Velocity (cm/sec)	V_0					
	V_{0x}					
	V_{0y}					
$t_{fli} \pm \Delta t_{fli} (sec)$	theo					
	exp					
	P.E.					
$h_{max} \pm \Delta h_{max} (cm)$	theo					
	exp					
	P.E.					
$R \pm \Delta R (cm)$	theo					
	exp					
	P.E.					

3.1 Example Calculations

- Calculations for velocity and its components of $\theta =$ ____:

- Theoretical value calculations for t_{fli} of $\theta =$ ____:

- Theoretical value calculations for h_{max} of $\theta =$ ____:

- Theoretical value calculations for R of $\theta =$ ____:

- Calculation for P.E. of ________ of $\theta =$ ____:

4 Conclusions

5 Notes

References

[1] Yunus Emre Akyol. Fizik laboratuvari 1 deneylerinin bilgisayar ortaminda simulasyonu. diploma thesis, Marmara University, 2017.