Experiment of Atwood's Machine

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1 Theoretical Background

One may study Newton's second law using a device known as Atwood's machine, shown in Figure (1). It consists of a pulley and two hanging masses. The difference in weight between the two hanging masses determines the net force acting on the system. This net force accelerates both of the hanging masses; the heavier mass is accelerated downward and the lighter mass is accelerated upward. This system is convenient for studying motion under constant acceleration because we can make the motion much slower, and easier to measure, than if we simply allowed objects to fall freely under the influence of gravity.

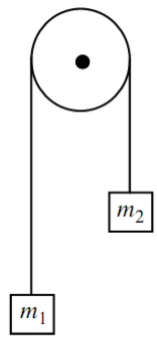

Figure 1: Atwood Machine

Since this experiment will be performed on airtable turned into an inclined plane, the Atwood's machine won't be hung up to a ceiling, but will be attached to the top of the inclined plane. This is shown in Figure (2).

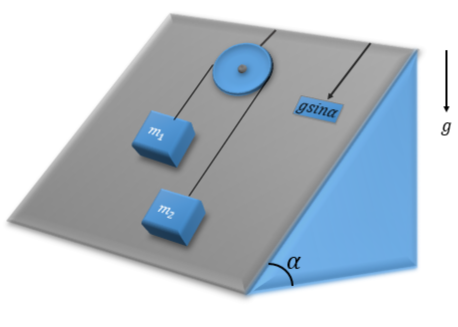

Figure 2: Atwood Machine on an inclined plane[1].

In the case of m_1 is less than m_2, m_1 accelerates upwards and m_2 accelerates downwards. The forces acting on the masses and their accelerations are given in Fig (3). Accordingly, the equations of motion for both masses can be written as follows;

$$\vec{F_1} = \vec{T_1} + m_1\vec{g}sin(\alpha) = -T_1\hat{\mathbf{i}} + m_1 gsin(\alpha)\hat{\mathbf{i}}, \quad (1)$$
$$\vec{F_2} = \vec{T_2} + m_2\vec{g}sin(\alpha) = -T_2\hat{\mathbf{i}} + m_2 gsin(\alpha)\hat{\mathbf{i}}, \quad (2)$$

where F_i is the force on i_th puck, T_i is the tension in ropes, m_i is mass of related puck, g is gravitational acceleration and α is the angle of inclined plane. According to the Newton's 2nd law of motion the net force acted upon a particle is equal to the multiplication of its mass and acceleration. Therefore, one can write;

$$\vec{F_1} = m_1\vec{a_1} = -m_1 a_1\hat{\mathbf{i}}, \quad (3)$$
$$\vec{F_2} = m_2\vec{a_2} = m_2 a_2\hat{\mathbf{i}}. \quad (4)$$

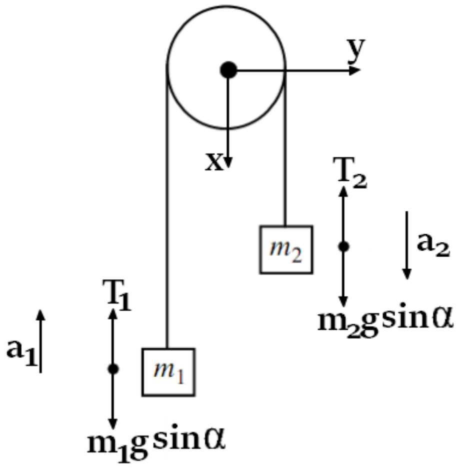

Figure 3: Force Diagram of Atwood Machine on an Inclined Plane.

Since the left-hand-sides of Eqn (1) and Eqn (3) are equal, the right-hand-sides of them should be equal as well. Then,

$$T_1 = m_1 gsin(\alpha) + m_1 a_1. \quad (5)$$

Similarly, from Eqn(2) and Eqn(4), T_2 might be obtained as follows,

$$T_2 = m_2 gsin(\alpha) - m_2 a_2. \quad (6)$$

If the rope of the pulley is inelastic the tensions and the accelerations for masses are the same in magnitude.

$$T_1 = T_2, \quad a_1 = a_2. \quad (7)$$

If Eqn(7) would evaluated with Eqn(5) and Eqn(6) one could get the acceleration for the system as;

$$a = \frac{m_2 - m_1}{m_2 + m_1} gsin(\alpha). \quad (8)$$

2 Procedure

2.1 Experimental Procedure

The experiment will be performed in two parts. One with constant mass difference and another with constant total mass.

2.1.1 Constant Total Mass

1. Turn the airtable into an inclined plane by placing its cylindrical part under its back foot.
2. Turn on the lab table and then the airtable.
3. By using only compressor's pedal, make sure the airtable is working.
4. If the airtable is working; turn it off. If not; contact with your lab instructor.
5. Place the carbon paper into the airtable and experiment sheet onto the carbon paper.
6. Hang the massless pulleys onto the airtable.
7. Disconnect the air hoses of the pucks. Weight them and note their mass values to Section 3. You may perform the weighting part only once.
8. Place the special hooks on both of the pucks and reconnect the air hoses.
9. Look at the 1st row of Table (1) and arrange the given masses of pucks.
10. Attach the hook of one puck to the one end of the special rope. Pass the rope around the pulleys and attach the other end of the rope to the hook of the other puck.
11. Place the pucks as shown in Figure (1). Make sure that the rope should be tense and the heavier puck should be placed higher.
12. Turn on the lab table and then the airtable.
13. By using pedals, run the airtable. If everything is set correctly, you should observe the heavier puck moving downwards and lighter puck moving upwards with an increasing speed.
14. Turn off the airtable.
15. Take the experiment paper down onto the experiment table and write mass values and frequency on the paper. Don't forget to label it with #1 and write the mass values and the frequency on it.
16. Repeat Step 7-15 for other rows on the Table (1) and label new experiment sheet with a label increased by 1.

2.1.2 Constant Mass Difference

1. Turn the airtable on.
2. By using only compressor's pedal, make sure the airtable is working.
3. If the airtable is working; turn it off. If not; contact with your lab instructor.
4. Place the carbon paper into the airtable and experiment sheet onto the carbon paper.
5. Hang the massless pulleys onto the airtable.
6. Disconnect the air hoses of the pucks. Weight them and note their mass values to Section 3. You may perform the weighting part only once.
7. Place the special hooks on both of the pucks and reconnect the air hoses.
8. Look at the 1st row of Table (2) and arrange the given masses of pucks.
9. Attach the hook of one puck to the one end of the special rope. Pass the rope around the pulleys and attach the other end of the rope to the hook of the other puck.
10. Place the pucks as shown in Figure (1). Make sure that the rope should be tense and the heavier puck should be placed higher.
11. Turn on the lab table and then the airtable.
12. By using pedals, run the airtable. If everything is set correctly, you should observe the heavier puck moving downwards and lighter puck moving upwards with an increasing speed.
13. Turn off the airtable.
14. Take the experiment paper down onto the experiment table and write mass values and frequency on the paper. Don't forget to label it with #5.
15. Repeat Step 6-14 for other rows on the Table (2) and label new experiment sheet with a label increased by 1.
16. **Make sure you tidy up your experiment table, turn your experiment table and the airtable off.**

2.2 Analysis Procedure

2.2.1 How to fill the tables

- Instruction for Table (1) and Table (2):
  1. Take the experiment sheet #1.
  2. Determine the first dot left by sparktimer and label it with 0.
  3. Label all the dots one by one until a suitable last one chosen by the student (let's say nth dot).
  4. Write the dot number into the related cell on Table (1).
  5. Using a ruler, measure the total distance between 0th and nth dots and write this value into the related cell on Table (1).
  6. Using the formula,

     $$t_n = \frac{n}{f}, \tag{9}$$

     calculate the time elapsed and write its square into the related cell on Table (1).

7. Using the distance formula for accelerated motion,

$$x(t) = \frac{1}{2}at^2, \quad \text{where } x_0 = 0 \text{ and } v_0 = 0, \tag{10}$$

calculate the experimental acceleration and write it into the related cell on Table (1).
8. Using the Eqn (8), calculate the theoretical acceleration and write it into the related cell on Table (1).
9. Using the percentage error formula,

$$P.E = \%\frac{|a_{the} - a_{exp}|}{a_{the}} * 100, \tag{11}$$

calculate the percentage error in the measured acceleration values and write it into the related cell on Table (1).
10. Repeat Step 6-14 for other rows on the Table (1) and Table (2).

- Instruction for Table (3):
  You will use same data from Table (2) to fill Table (3); you won't perform any new measurements or take any new data. Logarithmic calculations will be done only for Table (3).

  1. Calculate the total masses and write them into Table (3).
  2. Take the logarithms of total mass values and write them into Table (3).
  3. Copy the experimental acceleration values from Table (2) into Table (3).
  4. Calculate the logarithms of a_{exp} values and write them into Table (3).

2.2.2 Analysis for Constant Total Mass

1. Plot #1: Graph of a_{exp} vs. $(m_2 - m_1)$ with data from Table (1).
   - Plot a_{exp} vs. $m_2 - m_1$ graph and find the slope of it.
   - Its slope should give you the constant, $\frac{gsin(()\alpha)}{m_2+m_1}$.
   - By using numerical value of gravitational acceleration, the angle of the inclined plane and the slope, calculate the value of total mass.
   - Calculate the percentage error in total mass.

2.2.3 Analysis for Constant Mass Difference

2. Plot #2: Graph of a_{exp} vs. $(m_1 + m_2)^{-1}$ with data from Table (2).
   - Plot a_{exp} vs. $(m_2 + m_1)^{-1}$ graph and find the slope of it.
   - Its slope should give you the constant, $(m_2 - m_1)gsin(()\alpha)$.
   - By using numerical value of gravitational acceleration and the slope, calculate the value of mass difference.
   - Calculate the percentage error in the mass difference.
3. Plot #3: Graph of $\log(a_{exp})$ vs. $\log(m_1 + m_2)$ with data from Table (2).*
   - Plot $\log(a_{exp})$ vs. $\log(m_2 + m_1)$ graph and find the slope of it.
   - Its slope should give you -1.
   - Calculate the percentage error in the slope.
   - The y-intercept of the plot should yield $\log((m_2 - m_1)gsin(()\alpha))$.
   - By using numerical value of gravitational acceleration, the angle of the inclined plane and the y-intercept, calculate the value of mass difference.
   - Calculate the percentage error in the mass difference.

*If you wonder why to plot a logarithmic graph, please refer to Ref.[2].

3 Data & Results

Angle of inclined plane:

$$\alpha = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

Mass of the bare pucks:

$$m_1^{bare} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} g$$

$$m_2^{bare} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} g$$

Frequency of airtable:

$$f = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} Hz$$

3.1 Constant Total Mass

Fill the table below for part A.

Table 1: Table for part A. Total mass is $\sum m =$ ____________________g.

measurement	$m_1 \pm \Delta m_1$ (g)	$m_2 \pm \Delta m_2$ (g)	$\Delta m \pm \Delta(\Delta m)$ (g)	n	$x \pm \Delta x$ (cm)	$t^2 \pm \Delta(t^2)$ (sec^2)	$a_{exp} \pm \Delta a_{exp}$ (cm/sec^2)	a_{the} (cm/sec^2)	P.E.
1	650	850							%
2	625	875							%
3	600	900							%
4	575	925							%

3.2 Constant Mass Difference

Table 2: Data taken from experiment sheet in part B. Mass difference is $\Delta m =$ ______________g.

measurement	$m_1 \pm \Delta m_1$ (g)	$m_2 \pm \Delta m_2$ (g)	$(m_2+m_1)^{-1} \pm \Delta((m_2+m_1)^{-1})$ (g)	$x \pm \Delta x$ (cm)	$t^2 \pm \Delta(t^2)$ (sec^2)	$a_{exp} \pm \Delta a_{exp}$ (cm/sec^2)	a_{the} (cm/sec^2)	P.E.
5	575	715						%
6	585	725						%
7	595	735						%
8	605	745						%

Table 3: Logarithmic values of data of part B used in Plot#3. ($\Delta m =$ ______________g)

measurement	$m_1 \pm \Delta m_1$ (g)	$m_2 \pm \Delta m_2$ (g)	$(m_2+m_1) \pm \Delta(m_2+m_1)$ (g)	$\log(m_2+m_1)$	$a_{exp} \pm \Delta a_{exp}$ (cm/sec^2)	$\log(a_{exp})$
5	575	715				
6	585	725				
7	595	735				
8	605	745				

4 Conclusions

5 Notes

6 Another way to find acceleration of system

Now, let's follow a different way to find the acceleration of the system.
Let's think two objects separately and draw a free body diagram for each.
Don't forget that the mass of m_1 is smaller than the mass of m_2 when forming the free-body diagram. In line with this information, the direction of accelerations is found. Since m_1 is lighter, m_1 is accelerated to the upwards and m_2 is accelerated to the downwards.
For the mass of m_1, the free body diagram is as in Figure (4) and for the mass of m_2, the free-body diagram is the same as in Figure (5).

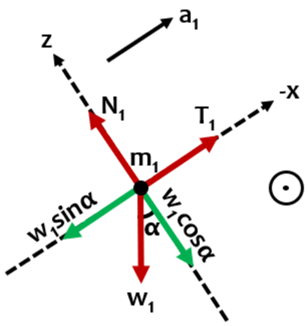

Figure 4: Free body diagram of m_1

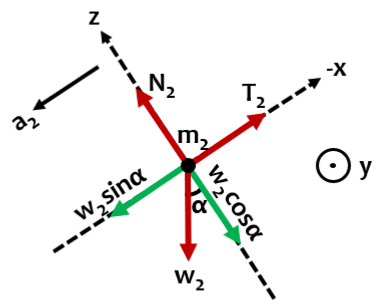

Figure 5: Free body diagram of m_2

In the free-body diagram, there are only the $forces$ of which the source is physical(real). Factors such as velocity and acceleration are placed *outside* the free body diagram. When creating a free-body diagram, this important detail should not be forgotten.

- Let us solve the free body diagram of mass m_1:

Since there is acceleration in the x-axis, Newton's second law of motion is applied.

$$\sum F_x = -T_1 + w_1 sin(\alpha) = -m_1 a_{1x} \tag{12}$$

There is no acceleration in the z-axis. In this case, $\vec{a_{1z}} = 0$. In line with this result, it can be said that there is a equilibrium state in z-axis. So, Newton's first law of motion is applied.

$$\sum F_y = N_1 - w_1 cos(\alpha) = 0 \tag{13}$$

- Let us solve the free body diagram of mass m_2:

Since there is acceleration in the x-axis, Newton's second law of motion is applied.

$$\sum F_x = -T_2 + w_2 sin(\alpha) = m_2 a_{2x} \tag{14}$$

There is no acceleration in the z-axis. In this case, $\vec{a_{2z}} = 0$. In line with this result, it can be said that there is a equilibrium state in z-axis. So, Newton's first law of motion is applied.

$$\sum F_y = N_2 - w_2 cos(\alpha) = 0 \tag{15}$$

If the rope of the pulley is inelastic the tensions and the accelerations for masses are the same in magnitude.

$$T_1 = T_2 \equiv T \tag{16}$$
$$a_{1x} = a_{2x} \equiv a \tag{17}$$

According to this information, the equations of motion can be rewritten as in (18) and (19).

$$-T + m_1 g sin(\alpha) = -m_1 a \tag{18}$$
$$-T + m_2 g sin(\alpha) = m_2 a \tag{19}$$

In order to find the acceleration, the T terms are destroyed and the two equations are transformed into a single equation as in (21).

$$m_1 g sin(\alpha) - m_2 g sin(\alpha) = -m_1 a - m_2 a \tag{20}$$

$$g sin(\alpha)(m_1 - m_2) = -a(m_1 + m_2) \tag{21}$$

As a result, the acceleration of the system is obtained as in (22).

$$a = \frac{m_2 - m_1}{m_2 + m_1} g sin(\alpha). \tag{22}$$

References

[1] Hakan Kubilay Karaagaç. Fizik laboratuvarı 1 deneylerinin bilgisayar ortamında simulasyonu. diploma thesis, Marmara University, 2017.

[2] Saba Arife Karakaş. Graphics guide. online published, “fzk.fef.marmara.edu.tr”, pg6, 10/2018.