Conservation of Linear Momentum and Energy

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1 Theoretical Background

1.1 Conservation of Momentum

The law of conservation of momentum states that,

> "For an isolated system, subject only to internal forces (forces between members of the system), the total linear momentum of the system is a constant; it does not change in time."[1]

Let us apply this law to the simplistic case of 2 body collision. According to the law, the momenta before and after the collision must be the same.

$$\vec{P}_{before} = \vec{P}_{after}, \tag{1}$$
$$\vec{P}_1(before) + \vec{P}_2(before) = \vec{P}_1(after) + \vec{P}_2(after), \tag{2}$$

where the linear momentum defined as,

$$\vec{P} = m\vec{V}, \tag{3}$$

In Eqn (3), m is the mass and $\vec{V}$ is the velocity of the bodies. We will use primes from now on to indicate the quantities after collision. The collision may be elastic or inelastic. In an elastic collision all the kinetic energy of the incoming particles reappears after the collision as kinetic energy but usually divided differently between the particles. In the usual inelastic collisions, part of the kinetic energy of the incoming particles appears after the collision as some form of internal excitation energy (such as heat) of one or more of the particles. It is important to realize that momentum conservation applies even to inelastic collisions, in which the kinetic energy is not conserved.
According to Newton's 2nd law of motion, the force on a body is defined as the change of its linear momentum over time.

$$\vec{F} = \frac{d\vec{P}}{dt}. \tag{4}$$

We assume that the bodies obey Newton's 3rd law of motion. Thus the force on the 1st body due to the 2nd body is,

$$\vec{F}_{12} = \frac{d\vec{P}_1}{dt} = \frac{d}{dt}(m_1\vec{V}_1), \tag{5}$$

and the force on the 2nd body due to the 1st body is,

$$\vec{F}_{21} = \frac{d\vec{P}_2}{dt} = \frac{d}{dt}(m_2\vec{V}_2), \tag{6}$$

and they are equal in magnitude and opposite in direction. Therefore,

$$\vec{F}_1 + \vec{F}_2 = 0, \tag{7}$$

$$= \frac{d\vec{P}_1}{dt} + \frac{d\vec{P}_2}{dt}, \tag{8}$$

$$= \frac{d}{dt}(\vec{P}_1 + \vec{P}_2). \tag{9}$$

And due to the definition of derivative, we get,

$$\vec{P}_1 + \vec{P}_2 = const. \tag{10}$$

Considering Eqn (2) one gets,

$$m_1\vec{V}_1 + m_2\vec{V}_2 = const. = m_1\vec{V'}_1 + m_2\vec{V'}_2, \tag{11}$$

where the primes indicate the quantities after collision. It should be emphasized that this equation is a vector equation and this equation does not let us to solve the collision problem uniquely. To find the unique solution, one needs additional information about the system. For some examples, this additional info provided by the scenario of the collision. It is also useful to use the conservation of energy in elastic collision examples.

Example: Elastic Collision in 1D

Consider an elastic collision of 2 bodies in 1 dimension represented in Fig (1). The total momentum before collision is,

$$\vec{P}_1 + \vec{P}_2 = m_1\vec{V}_1 + m_2\vec{V}_2 = m_1V_1\hat{\mathbf{i}} - m_2V_2\hat{\mathbf{i}}, \tag{12}$$

where the x coordinate is chosen as the right side of the reader. Similarly the total momentum after the collision is,

$$\vec{P'}_1 + \vec{P'}_2 = m_1\vec{V'}_1 + m_2\vec{V}_2' = -m_1V_1'\hat{\mathbf{i}} + m_2V_2'\hat{\mathbf{i}}. \tag{13}$$

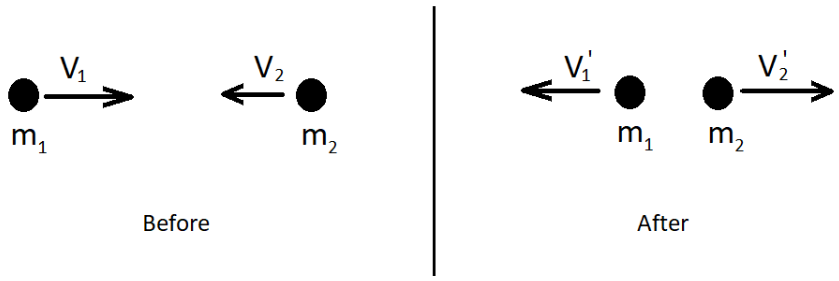

Figure 1: Before and after of an elastic collision in 1D.

According to Eqn (11),

$$m_1V_1 - m_2V_2 = -m_1V_1' + m_2V_2' \tag{14}$$

One can write infinitely many solutions to this equation, because for the same initial mass and velocity values, there are infinitely many different final velocities. Some of these solutions are listed in Table (1). Therefore some additional information is needed to solve this problem uniquely. To do so, let's assume that the masses of the bodies are same and that in the collision, the first particle is brought to rest.

$$m_1 = m_2 \equiv m, \quad V_1' = 0. \tag{15}$$

Then using Eqn (14) yields,

$$V_2' = V_1 - V_2. \tag{16}$$

Notice that this still is not a unique solution. But if we consider 1 more constriction such as the initial state of 2nd body would be at rest, then we get a unique solution as follows,

$$V_2' = V_1, \quad V_1' = 0. \tag{17}$$

This solution is unique because the resultant velocities are determined by the initial quantities with only 1 way.

	V_1'	V_2'
Solutions for $m_1 = m_2 \equiv m$ 4*and $V_1 = V_2 \equiv V$	V	V
	$V/2$	$V/2$
	$4V$	$4V$
	...	...
Solutions for $m_1 = 2m_2 \equiv 2m$ 5*and $3V_1 = V_2 \equiv 3V$	V	V
	$V/2$	0
	$3V/2$	$2V$
	$2V$	$3V$
	...	...

Table 1: Some solutions for Eqn (14). Watch that there are more than 1 final value pairs for the same initial values.

Example: Inelastic Collision in 1D

Inelastic 2-body collision in 1D is shown in Figure (2). Using the conservation of momentum law, we get,

$$(m_1 V_1 - m_2 V_2)\hat{\mathbf{\imath}} = (m_1 + m_2)\vec{V}' \tag{18}$$

Again, this equation does not have a unique solution. To find a unique solution, we may consider some different conditions. Or we may consider another fundamental conservation law of physics, the conservation of energy.

1.2 Conservation of Energy

The law of conservation of energy states that for a system of particles with interactions not explicitly[1] dependent on the time, the total energy of the system is constant.We accept this result as a very well established experimental fact. More specifically, the law tells us there exists some scalar function [such as the function $Mv^2/2$] of the positions and velocities of the constituent particles that is invariant with respect to a change in time, provided there is no explicit change in the interaction forces during the time interval considered[1].

[1]Consider the system with the particles permanently frozen in place: then a force that depends on time IS said to depend explicitly on time

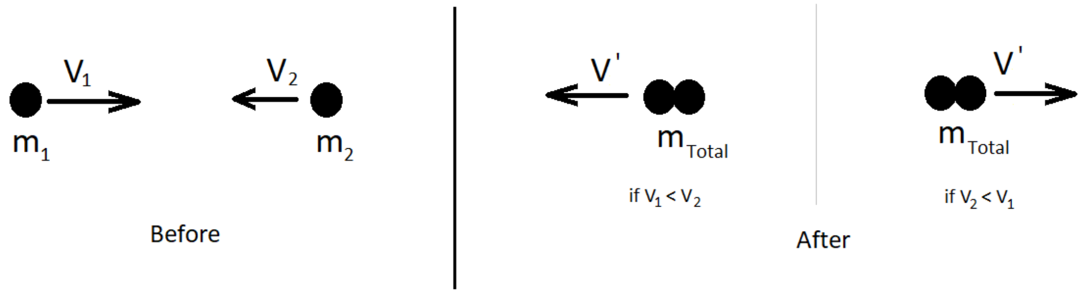

Figure 2: Inelastic collision in 1D. Direction of the final velocity depends on the magnitudes of the initial velocities.

According to Eqn (3) an object at rest will have zero linear momentum. On the other hand, the net external force equals the change in momentum of a system divided by the time over which it changes[2],

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}. \tag{19}$$

Eqn (19) is called as "Newton's 2nd law of motion". Eqn (19) indicates that if no net external force applied on a system then its total linear momentum does not change over time.

$$\vec{F}_{ext} = 0 \qquad \Rightarrow \qquad \frac{d\vec{P}_{total}}{dt} = 0. \tag{20}$$

If the system consists of 2 different particles then the total momentum of the system would be,

$$\vec{P}_{total} = \vec{P}_1 + \vec{P}_2. \tag{21}$$

When there is no external force acting on this system, the momentum change for one particle would be,

$$\frac{d}{dt}(\vec{P}_1 + \vec{P}_2) = 0 \qquad \Rightarrow \qquad \dot{\vec{P}}_1 = -\dot{\vec{P}}_2. \tag{22}$$

Here, the dot over momenta represents the time derivative.

2 Procedure

2.1 Experimental Procedure

2.1.1 Part A: Elastic Collision

1. Turn on the lab table and then the airtable.
2. By using only compressor's pedal, make sure the airtable is working.
3. If the airtable is working; turn it off. If not; contact with your lab instructor.
4. Place the carbon paper into the airtable and experiment sheet onto the carbon paper.
5. Set the frequency of the airtable to 20 Hz and write this value down in Section 3.2.
6. Turn on the experiment table and the airtable.
7. Place one of the puck at lower right corner inside of the airtable.
8. Place the other puck at lower left corner inside of the airtable.
9. Push both of the pedals.

10. Give the pucks a small push with the directions shown in Fig 3. They should make a motion with constant velocity. Be careful about that your hand should touch the puck for a really small amount of time. The pucks should collide at the center of airtable and shold keep moving in opposite x-direction and same y-direction. The path you should see on the experiment paper is shown in Fig 4.
11. Just as the pucks reach to the end of airtable remove your hands from pucks.
12. Turn the airtable and the experiment table off.
13. Move the experiment sheet from the airtable to the experiment table.

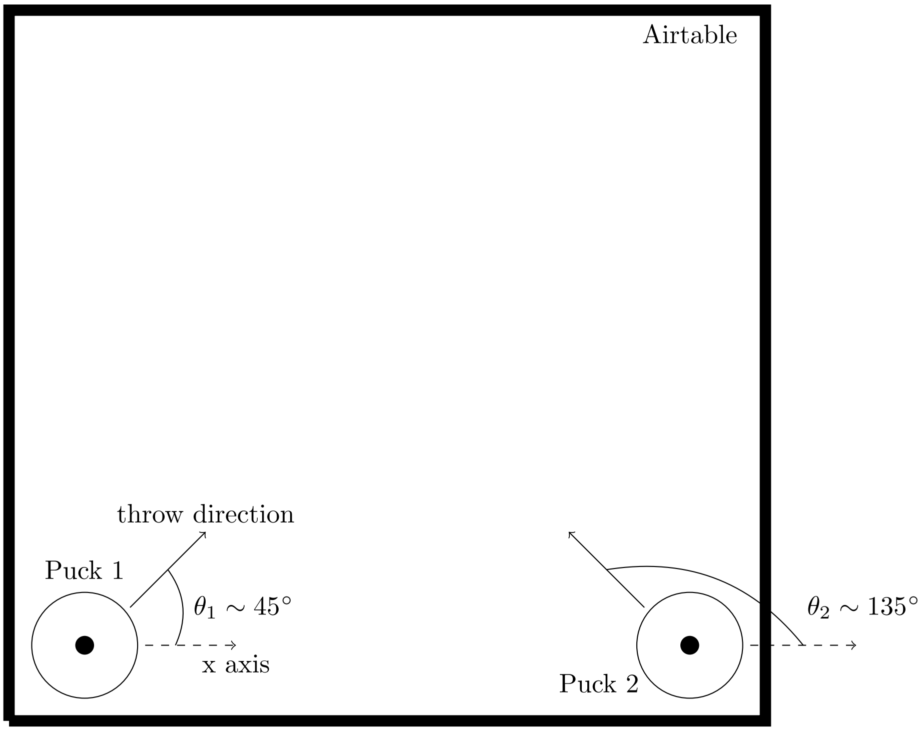

Figure 3: Airtable from top-view. You should push the pucks as shown in this figure.

2.1.2 Part B: Inelastic Collision

1. Turn on the lab table and then the airtable.
2. By using only compressor's pedal, make sure the airtable is working.
3. If the airtable is working; turn it off. If not; contact with your lab instructor.
4. Place the carbon paper into the airtable and experiment sheet onto the carbon paper.
5. Set the frequency of the airtable to 20 Hz and write this value down in Section 3.2.
6. Surround the pucks with hook-and-loop fasteners.
7. Turn on the experiment table and the airtable.
8. Place one of the puck at lower right corner inside of the airtable.
9. Place the other puck at lower left corner inside of the airtable.
10. Push both of the pedals.

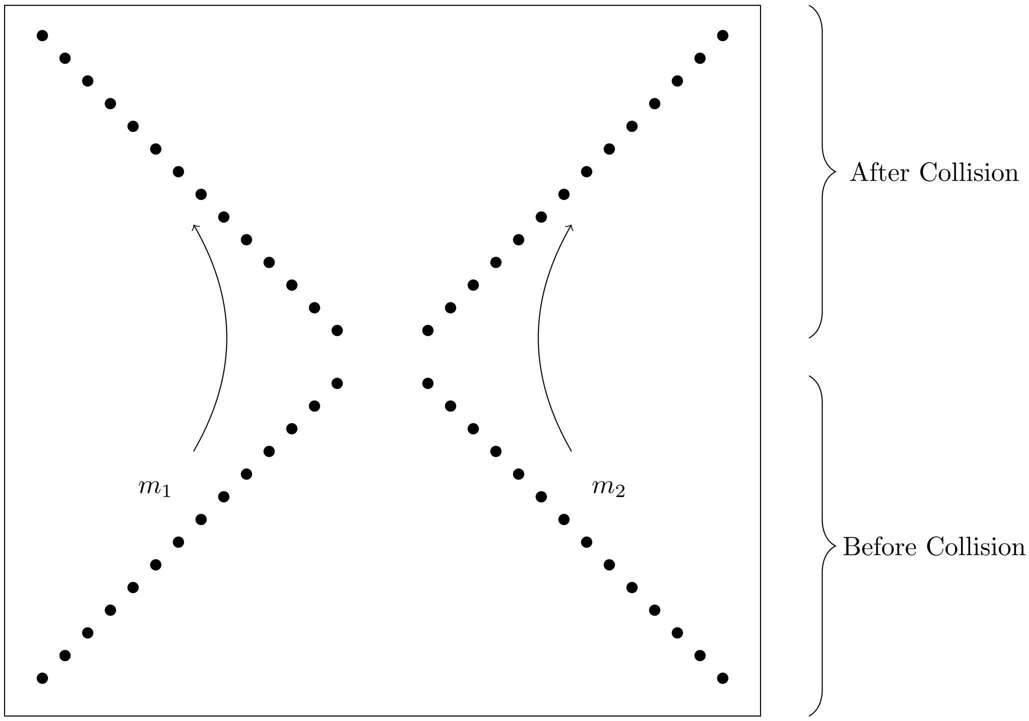

Figure 4: Experimental sheet of Part A, elastic collision.

11. Give the pucks a small push with the directions shown in Fig 3. They should make a motion with constant velocity. Be careful about that your hand should touch the puck for a really small amount of time. The pucks should collide at the center of airtable, they should stick together and keep moving in same y-direction together. The path you should see on the experiment paper is shown in Fig 5.
12. Just as the pucks reach to the end of airtable remove your hands from pucks.
13. Turn the airtable and the experiment table off.
14. Move the experiment sheet from the airtable to the experiment table.

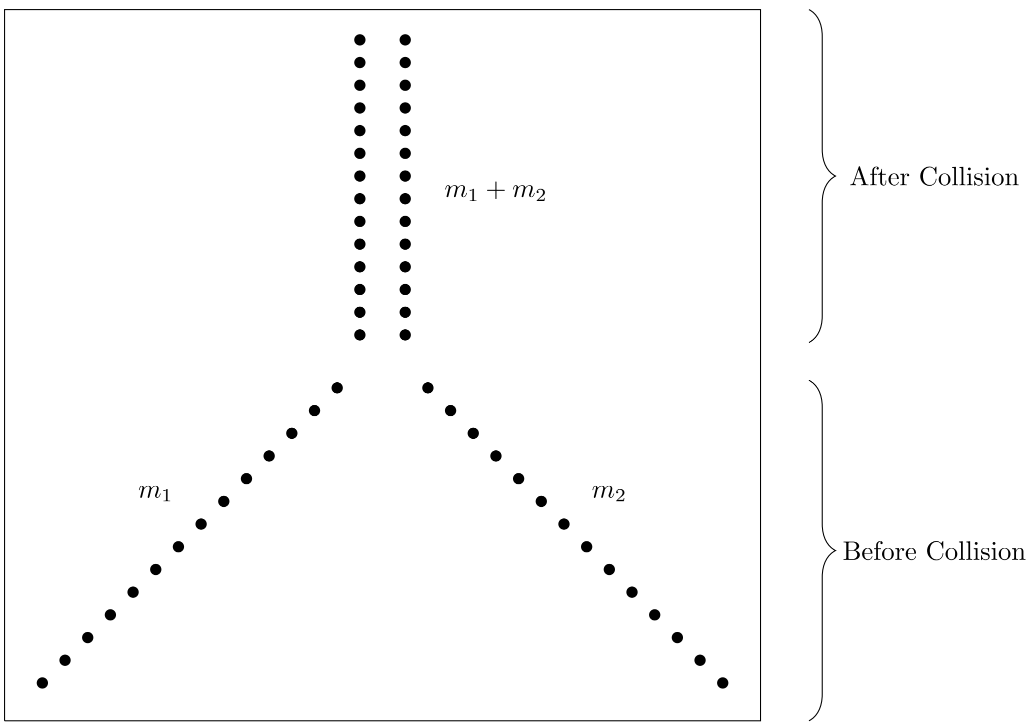

Figure 5: Experimental sheet of Part B, inelastic collision.

2.2 Analysis Procedure

2.2.1 Part A: Elastic Collision

1. Measure the masses of pucks, m_1 and m_2. And write these values down on Section 3.2.
2. Mark the first two dots of before and after collision for each pucks. Fig 6 shows how to choose the dots and mark them.
3. Measure x_1, x_2, x'_1 and x'_2 and write them down in Section 3.2.
4. Calculate the total time spent for one interval in your experiment. Write this value as t_1, t_2, t'_1 and t'_2 in Section 3.2.
5. Calculate the speed values, v_1, v_2, v'_1 and v'_2, and write them down in Section 3.2. Here v_1 is the speed of Puck 1 before collision, v_2 is the speed for Puck 2 before collision, v'_1 is the speed for Puck 1 after collision and v'_2 is the speed for Puck 2 after collision.
6. By using a goniometer[2], measure the positive-definite angles, θ_1, θ_2, θ'_1 and θ'_2. Write them down in Section 3.2.
7. Calculate the kinetic energies for pucks separately, for before and after the collision. Use the notation as follows: K_1 for kinetic energy of Puck 1 before collision, K_2 for kinetic energy of Puck 2 before collision, K'_1 for kinetic energy of Puck 1 after collision, K'_2 for kinetic energy of Puck 2 after collision. Do not forget to write your findings in Section 3.2.
8. Calculate the total kinetic energies before ($K = K_1 + K_2$) and after ($K' = K'_1 + K'_2$) the collision. Write them down in Section 3.2.

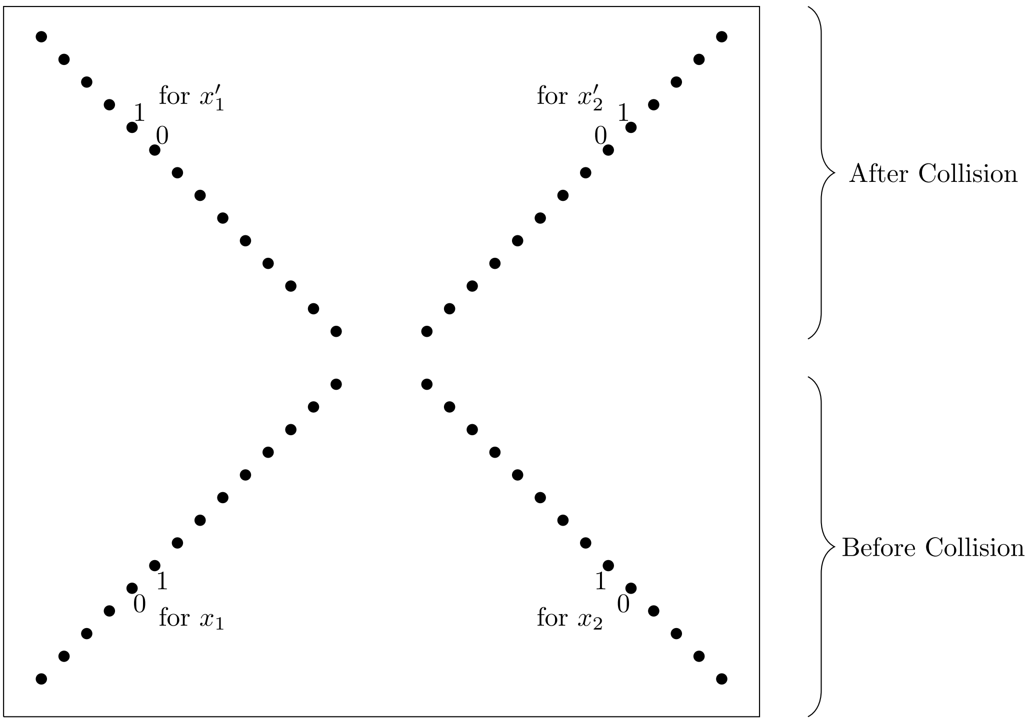

Figure 6: How to choose and mark necessary dots for analysis of Part A.

9 Calculate the momentum components for each of the puck and for before and after the collision separately. Formulas are given in Section 3.2. Write them in Section 3.2.

10 Calculate the total momentum components for before and after the collision. Write them in Section 3.2.

[2]gönye

2.2.2 Part B: Inelastic Collision

1. Measure the masses of pucks, m_1 and m_2. And write these values down on Section 3.2.
2. Mark the first two dots of before and after collision for each pucks. Fig 6 shows how to choose the dots and mark them.
3. Measure x_1, x_2 and x' and write them down in Section 3.2.
4. Calculate the total time spent for one interval in your experiment. Write this value as t_1, t_2 and t' in Section 3.2.
5. Calculate the speed values, v_1, v_2 and v', and write them down in Section 3.2. Here v_1 is the speed of Puck 1 before collision, v_2 is the speed for Puck 2 before collision and v' is the speed for pucks that are spliced together after collision.
6. By using a goniometer[3], measure the positive-definite angles, θ_1, θ_2 and θ'. Write them down in Section 3.2.
7. Calculate the kinetic energies for pucks separately, for before and after the collision. Use the notation as follows: K_1 for kinetic energy of Puck 1 before collision, K_2 for kinetic energy of Puck 2 before collision, K' for kinetic energy of pucks after collision. Do not forget to write your findings in Section 3.2.
8. Calculate the total kinetic energies before ($K = K_1 + K_2$) and after (K') the collision. Write them down in Section 3.2.

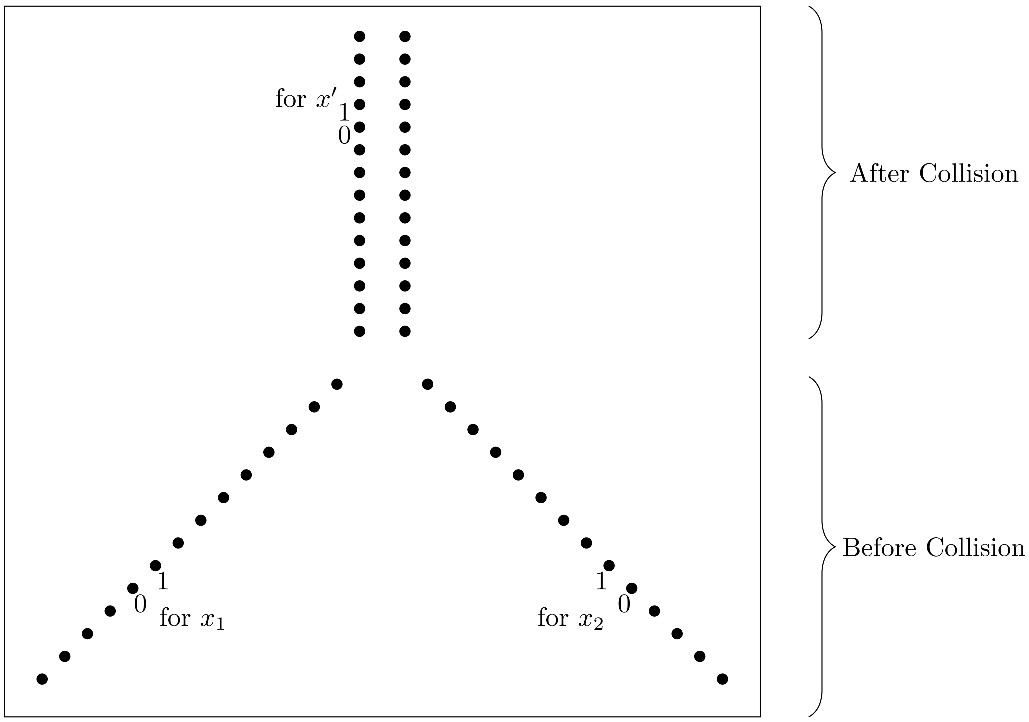

Figure 7: How to choose and mark necessary dots for analysis of Part B.

[3] gönye

9 Calculate the momentum components for each of the puck and for before the collision separately. Formulas are given as follows. Write them in Section 3.2.

$$P_{1,x} = m_1 v_1 \cos(\theta_1), \tag{23}$$
$$P_{1,y} = m_1 v_1 \sin(\theta_1), \tag{24}$$
$$P_{2,x} = m_2 v_2 \cos(\theta_2), \tag{25}$$
$$P_{2,y} = m_2 v_2 \sin(\theta_2). \tag{26}$$

10 Calculate the total momentum components for before and after the collision. Write them in Section 3.2.

$$P_x = P_{1,x} + P_{2,x}, \tag{27}$$
$$P_y = P_{1,y} + P_{2,y}, \tag{28}$$
$$P'_x = (m_1 + m_2) v' \cos(\theta'), \tag{29}$$
$$P'_y = (m_1 + m_2) v' \sin(\theta'), \tag{30}$$
$$\tag{31}$$

3 Data & Analysis

3.1 Part A: Elastic Collision

- Measurements for m_1 and m_2:

- Measurements for x_1, x_2, x'_1 and x'_2:

- Calculations for t_1, t_2, t'_1 and t'_2:

- Calculations for v_1, v_2, v'_1 and v'_2:

- Calculations for K_1, K_2, K'_1 and K'_2:

- Calculations for $K = K_1 + K_2$ and $K' = K'_1 + K'_2$:

- Calculations for $P_{1,x}$ and $P_{2,x}$:

- Calculations for $P'_{1,x}$ and $P'_{2,x}$:

- Calculations for $P_x = P_{1,x} + P_{2,x}$ and $P'_x = P'_{1,x} + P'_{2,x}$:

- Calculations for $P_{1,y}$ and $P_{2,y}$:

- Calculations for $P'_{1,y}$ and $P'_{2,y}$:

- Calculations for $P_y = P_{1,y} + P_{2,y}$ and $P'_y = P'_{1,y} + P'_{2,y}$:

3.2 Part B: Inelastic Collision

- Measurements for m_1 and m_2:

- Measurements for x_1, x_2 and x':

- Calculations for t_1, t_2 and t':

- Calculations for v_1, v_2 and v':

- Calculations for K_1 and K_2:

- Calculations for $K = K_1 + K_2$ and K':

- Calculations for $P_{1,x}$ and $P_{2,x}$:

- Calculations for $P_x = P_{1,x} + P_{2,x}$:

- Calculations for P'_x:

- Calculations for $P_{1,y}$ and $P_{2,y}$:

- Calculations for $P_y = P_{1,y} + P_{2,y}$:

- Calculations for P'_y:

4 Conclusions

5 Notes

References

[1] MalVin A Helmholz A Carl Moyer Burton J. Knight. Walter D, Ruderman. *Berkeley Physics Course v.1 Mechanics.* McGraw-Hili, Inc., 1973.

[2] Open Stax College. Linear momentum and force - collage physics, 2018. [Online; accessed 31-October-2018].