Capacitors

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1 Theoretical Background

A capacitor is a passive two-terminal electronic component that stores electrical energy in an electric field. The effect of a capacitor is known as capacitance.[2]

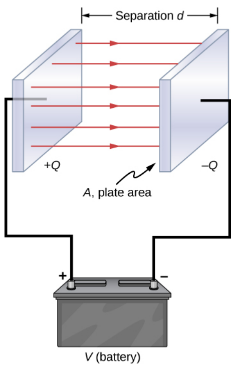

Figure 1: Parallel plate capacitor[1].

The physical form and construction of practical capacitors vary widely and many capacitor types are in common use. Most capacitors contain at least two electrical conductors often in the form of metallic plates or surfaces separated by a dielectric medium. A conductor may be a foil, thin film, sintered bead of metal, or an electrolyte. The nonconducting dielectric acts to increase the capacitor's charge capacity. Materials commonly used as dielectrics include glass, ceramic, plastic film, paper, mica, air, and oxide layers. Capacitors are widely used as parts of electrical circuits in many common electrical devices. Unlike a resistor, an ideal capacitor does not dissipate energy.[2] The energy that capacitor reserves depends on the capacitance value and the charge on the capacitor. The capacitance value of a parallel plate capacitor is,

$$C = \epsilon_0 \frac{A}{d}, \tag{1}$$

where C, ϵ_0, A and d are the capacitance, electrical constant, area of one plate and separation of two plates respectively. Electrical potential between two plates are determined by the capacitance as,

$$V = \frac{Q}{C}, \tag{2}$$

where V and Q are the electrical potential between plates and the absolute value of total charge on one plate. When two conductors experience a potential difference, for example, when a capacitor is attached across a battery, an electric field develops across the dielectric, causing a net positive charge to collect on one plate and net negative charge to collect on the other plate. No current actually flows through the dielectric. However, there is a flow of charge through the source circuit. If the condition is maintained sufficiently long, the current through the source circuit ceases over time.[2] The behaviour of current through and voltage over the capacitor can be determined by simple formulas. In the lecture we will connect the capacitor with a resistor in serial. Thus the source voltage is the sum of voltages over these two passive circuit elements,

$$V_s = V_R + V_C, \tag{3}$$

where subindices s, R and C denotes source, resistor and capacitor respectively. By using Ohm's law and Eqn. (2) one will get,

$$V_s = IR + \frac{Q}{C} \qquad \Rightarrow \qquad \frac{V_s}{R} = I + \frac{Q}{RC}. \tag{4}$$

Remember that the charge over the capacitor changes in time and thus the charge and the current is a function of time. Therefore we should write,

$$\frac{V_s}{R} = I(t) + \frac{Q(t)}{RC}. \tag{5}$$

When $t = 0$, there is no charge on the capacitor and we will denote $I(t = 0)$ as I_0 as the initial current at the beginning.

$$\frac{V_s}{R} = I_0. \tag{6}$$

To understand how the charge and the current changes over time, we will derivate the Eqn. (5) as follows,

$$\underbrace{\frac{d}{dt}\frac{V_s}{R}}_{=0} = \frac{d}{dt}\left(I(t) + \frac{Q(t)}{RC}\right) = \frac{dI}{dt} + \frac{1}{RC}\frac{dQ}{dt}. \tag{7}$$

Please remember that the charge derivation over time yields the current in the circuit. Then,

$$\begin{aligned}
0 &= \frac{dI}{dt} + \frac{1}{RC}I \\
\frac{1}{I}\frac{dI}{dt} &= -\frac{1}{RC} \\
\frac{1}{I}\frac{dI}{dt}dt &= -\frac{1}{RC}dt \\
\underbrace{\int_{t=0}^{t}\frac{1}{I}\frac{dI}{dt}dt}_{} &= -\int_{t=0}^{t}\frac{dt}{RC} \\
ln(I(t))|_{t=0}^{t} &= -\frac{t}{RC}|_{t=0}^{t} \\
ln(I(t)) - ln(I_0) &= -\frac{t}{RC} \\
ln(\frac{I(t)}{I_0}) &= -\frac{t}{RC} \\
I(t) &= I_0 e^{-t/RC} \qquad (8)
\end{aligned}$$

To obtain voltage of condansator one may use Eqn. (2),

$$Q = CV \qquad \Rightarrow \qquad I(t) = C\frac{dV}{dt} \tag{9}$$

By writing Eqn. (8) into above equation and taking the necessary integration one may get the voltage of contansator while charging as,

$$V_C(t) = V_s(1 - e^{-t/RC}), \tag{10}$$

where $V_s = I_0 R$. Eqn. (8) and Eqn. (10) are the current and voltage during the charging process. One may get the current and voltage during the discharge process using the similar way as,

$$\begin{aligned}
I(t) &= I_0 e^{-t/RC}, \qquad (11) \\
V_C(t) &= V_s e^{-t/RC}. \qquad (12)
\end{aligned}$$

You need to notify that for discharging process at $t = 0$, $I(t = 0) = I_0$ and $V_C(t = 0) = V_s$.[1]

In voltage and current formulas during both charging and discharging, one constant draws attention. $\tau = RC$ is the time constant of the circuit and it determines the time behaviour of the capacitor. One may use this constant to find how many percentage of the capacitance is charged/discharged at a specific time. You may check Table (1) to see the power of this constant. To understand it let's check first two specific time points.

- During Charge:

$$t = 0, \quad I(t = 0) = I_0 \underbrace{e^{0/\tau}}_{=1} = I_0, \quad V_C(t = 0) = V_s(1 - e^{0/\tau}) = 0$$

$$t = \tau, \quad I(t = \tau) = I_0 \underbrace{e^{-\tau/\tau}}_{e^{-1}=0.37} = 0.37 I_0, \quad V_C(t = 0) = V_s(1 - e^{-\tau/\tau}) = 0.63 V_s$$

$$t = 2\tau, \quad I(t = 2\tau) = I_0 \underbrace{e^{-2\tau/\tau}}_{e^{-1}=0.14} = 0.14 I_0, \quad V_C(t = 0) = V_s(1 - e^{-2\tau/\tau}) = 0.86 V_s$$

$$t = 3\tau, \quad ...$$

[1]Because the condansator is full.

As you can see from the above equations, the capacitor is %63 charged at $t = \tau$ second and it is %86 charged at $t = 2\tau$ second. To charge a capacitor fully, 6-7τ duration should pass. And note that the current ceases over time. You may check that at $t = 7\tau$ the current is nearly zero.

- During Disharge:

$$t = 0, \quad I(t=0) = I_0 \underbrace{e^{0/\tau}}_{=1} = I_0, \quad V_C(t=0) = V_s e^{0/\tau} = 0$$

$$t = \tau, \quad I(t=\tau) = I_0 \underbrace{e^{-\tau/\tau}}_{e^{-1}=0.37} = 0.37 I_0, \quad V_C(t=0) = V_s e^{-\tau/\tau} = 0.37 V_s$$

$$t = 2\tau, \quad I(t=2\tau) = I_0 \underbrace{e^{-2\tau/\tau}}_{e^{-1}=0.14} = 0.14 I_0, \quad V_C(t=0) = V_s e^{-2\tau/\tau} = 0.14 V_s$$

$$t = 3\tau, \quad ...$$

During discharge both current and voltage over capacitor decreases over time. At $t = \tau$, capacitor has lost %63 voltage of its initial voltage and at $t = 2\tau$, it has lost %86 voltage of its initial voltage.

t	Charge		Discharge	
	V_C	I	V_C	I
0	0%	100%	100%	100%
τ	63%	37%	37%	37%
2τ	86%	14%	14%	14%
3τ	95%	5%	5%	5%
4τ	98%	2%	2%	2%
5τ	99.3%	0.7%	0.7%	0.7%
6τ	99.8%	0.2%	0.2%	0.2%
7τ	99.99%	0.01%	0.01%	0.01%

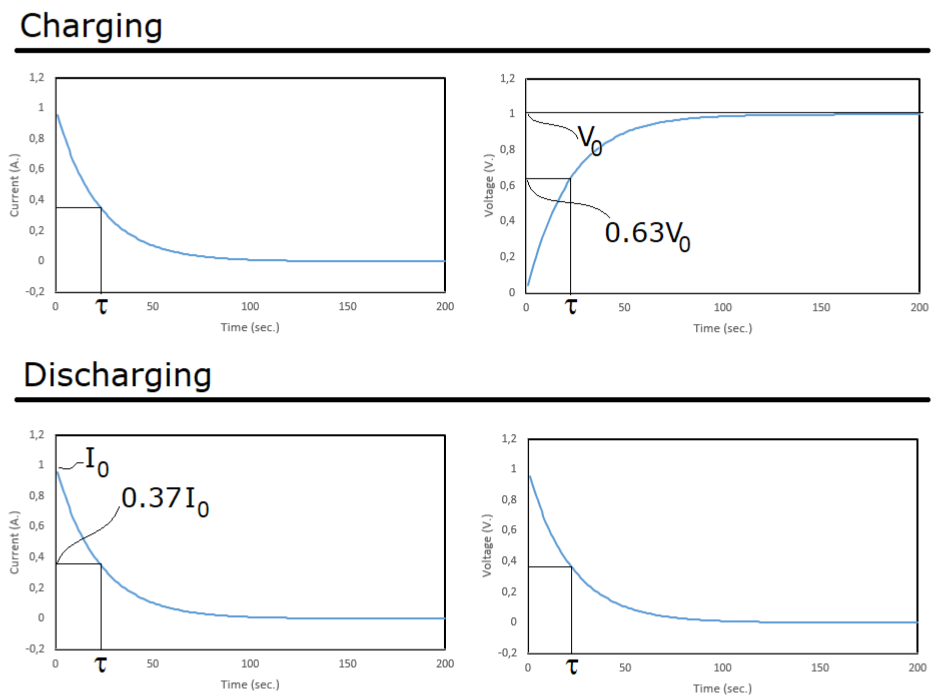

Figure 2: Plots for current vs. time and voltage vs. time during both charging and discharging.

2 Procedure

2.1 Experimental Procedure

1. By using the power supply and a multimeter, adapt the source voltage 5V and immediately turn it off. Perform this step very quickly!

2. Set the circuit given in Fig (3) with a and $10k\Omega$ resistor and $2200\mu F$ capacitor.

3. Calculate the time constant and seven times of the time constant of the circuit. Write these values down in Section 3.

4. Place a chronometer next to the voltmeter and ammeter. By using small pieces of paper, write the names of the multimeters (write which one is the voltmeter and which one is the ammeter.)

5. By using your phone's video feature, start recording the circuit while source is turned off and read the specifics of the circuit verbally so that the video can record your voice.[2] DO NOT STOP RECORDING.

6. Turn on the chronometer and the source simultaneously while recording. Record for approximately 4min. of circuit charge. DO NOT STOP RECORDING.But reset your chronometer.

7. After approximately 4min. of charging, turn on the switch 1, turn off the switch 2 and turn on the chronometer simultaneously.

8. Keep recording until you see ≈ 0 at the voltmeter. When you see zero, you may stop recording.

9. Repeat steps 2-8 with $1000\mu F$ capacitor and $10k$ resistor as Part B.

10. Repeat steps 1-8 with $10V$ of source voltage as Part C.

[2]You should read the resistance, capacitance and source voltage values.

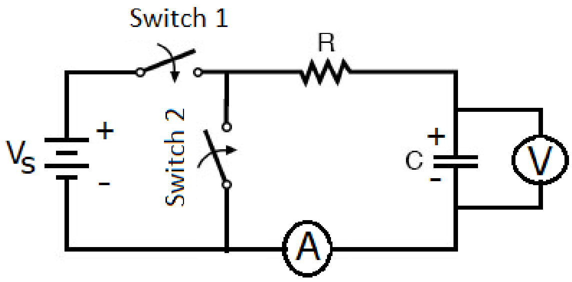

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Figure 3: Circuit for the experiment.

2.1.1 Analysis Procedure

1. By watching the recordings fill all the tables in Section 3.

2. Plot the $V_C - t$ and $I - t$ graphs for both charging and discharging in all parts. You should plot 12 graphs in total.

3. Using the source voltages, calculate 63% of it to analyze $V_C - t$ plots during charging. Write them in Section 3.

4. Find the $0.63V_s$ value at the $y - axis$ and read the corresponding time value from $x - axis$. This is your experimental τ value for $V_c - t$ plots during charging.

5. Using the source voltages, calculate 37% of it to analyze $V_C - t$ plots during discharging. Write them in Section 3.

6. Find the $0.37V_s$ value at the $y - axis$ and read the corresponding time value from $x - axis$. This is your experimental τ value for $V_c - t$ plots during discharging

7. Using the source voltages, first calculate the initial current values and then calculate 37% of it to analyze $I - t$ plots during both charging and discharging. Write them in Section 3.

8. Find the $0.37I_0$ value at the $y - axis$ and read the corresponding time value from $x - axis$. This is your experimental τ value for $I - t$ plots during both charging and discharging.

<u>NOTE:</u> You may check Fig. (2) to grasp the details about analysis.

3 Data & Analysis

3.1 Part A: $C = 2200\mu F$, $R = 10k\Omega$, $V_s = 5V$

Calculate the following values in detail.

$$\tau = \text{--} \quad (13)$$

$$7\tau = \text{--} \quad (14)$$

Charging

Table 1: Table of time, voltage and current values during charging for $C = 2200\mu F$, $R = 10k\Omega$, $V_s = 5V$.

#	t (*sec.*)	I (---)	V_C (V)	#	t (*sec.*)	I (---)	V_C (V)	#	t (*sec.*)	I (---)	V_C (V)
1	1			21	21			41	90		
2	2			22	22			42	95		
3	3			23	24			43	100		
4	4			24	26			44	105		
5	5			25	28			45	110		
6	6			26	30			46	115		
7	7			27	32			47	120		
8	8			28	34			48	125		
9	9			29	36			49	130		
10	10			30	38			50	135		
11	11			31	40			51	140		
12	12			32	45			52	145		

13	13			33	50			53	150		
14	14			34	55			54	155		
15	15			35	60			55	160		
16	16			36	65			56	165		
17	17			37	70			57	170		
18	18			38	75			58	180		
19	19			39	80			59	190		
20	20			40	85			60	200		

- Analysis details of $V_C - t$ plot during charging.

$$63\% \text{ of } V_s = \text{------------------------------} \quad (15)$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{------------------------------} \quad (16)$$

$$\tau_{the} = \text{------------------------------} \quad (17)$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \%\text{--------------------} \quad (18)$$

- Analysis details of $I - t$ plot during charging.

$$I_0 = \text{---} \quad (19)$$

$$37\% \text{ of } I_0 = \text{---} \quad (20)$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{---} \quad (21)$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \%\text{-------------------} \quad (22)$$

Disharging

Table 2: Table of time, voltage and current values during discharging for $C = 2200\mu F$, $R = 10k\Omega$, $V_s = 5V$.

#	t (*sec.*)	I (___)	V_C (V)	#	t (*sec.*)	I (___)	V_C (V)	#	t (*sec.*)	I (___)	V_C (V)
1	1			21	21			41	90		
2	2			22	22			42	95		
3	3			23	24			43	100		
4	4			24	26			44	105		
5	5			25	28			45	110		

6	6			26	30			46	115		
7	7			27	32			47	120		
8	8			28	34			48	125		
9	9			29	36			49	130		
10	10			30	38			50	135		
11	11			31	40			51	140		
12	12			32	45			52	145		
13	13			33	50			53	150		
14	14			34	55			54	155		
15	15			35	60			55	160		
16	16			36	65			56	165		
17	17			37	70			57	170		
18	18			38	75			58	180		
19	19			39	80			59	190		
20	20			40	85			60	200		

- Analysis details of $V_C - t$ plot during discharging.

$$37\% \text{ of } V_s = \text{...} \tag{23}$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{...} \tag{24}$$

$$\tau_{the} = \text{...} \tag{25}$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \%\text{..............................} \tag{26}$$

- Analysis details of $I - t$ plot during discharging.

$$I_0 = \text{...} \tag{27}$$

$$37\% \text{ of } I_0 = \text{...} \tag{28}$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{...} \tag{29}$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \%\text{..............................} \tag{30}$$

3.2 Part B: $C = 1000\mu F$, $R = 10k\Omega$, $V_s = 5V$

Calculate the following values in detail.

$$\tau = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \tag{31}$$

$$7\tau = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \tag{32}$$

Charging

Table 3: Table of time, voltage and current values during charging for $C = 1000\mu F$, $R = 10k\Omega$, $V_s = 5V$.

#	t (*sec.*)	I (___)	V_C (V)	#	t (*sec.*)	I (___)	V_C (V)	#	t (*sec.*)	I (___)	V_C (V)
1	1			21	21			41	90		
2	2			22	22			42	95		
3	3			23	24			43	100		
4	4			24	26			44	105		
5	5			25	28			45	110		
6	6			26	30			46	115		
7	7			27	32			47	120		
8	8			28	34			48	125		
9	9			29	36			49	130		
10	10			30	38			50	135		
11	11			31	40			51	140		
12	12			32	45			52	145		

13	13			33	50			53	150		
14	14			34	55			54	155		
15	15			35	60			55	160		
16	16			36	65			56	165		
17	17			37	70			57	170		
18	18			38	75			58	180		
19	19			39	80			59	190		
20	20			40	85			60	200		

- Analysis details of $V_C - t$ plot during charging.

$$63\% \text{ of } V_s = \text{------------------------------} \quad (33)$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{------------------------------} \quad (34)$$

$$\tau_{the} = \text{------------------------------} \quad (35)$$

$$\text{P.E.} = \%\frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \%\text{------------------} \quad (36)$$

- Analysis details of $I-t$ plot during charging.

$$I_0 = \text{--} \quad (37)$$

$$37\% \text{ of } I_0 = \text{--} \quad (38)$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{--} \quad (39)$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \%\text{------------------------} \quad (40)$$

Discharging

Table 4: Table of time, voltage and current values during discharging for $C = 1000\mu F$, $R = 10k\Omega$, $V_s = 5V$.

#	t (*sec.*)	I (---)	V_C (V)	#	t (*sec.*)	I (---)	V_C (V)	#	t (*sec.*)	I (---)	V_C (V)
1	1			21	21			41	90		
2	2			22	22			42	95		
3	3			23	24			43	100		
4	4			24	26			44	105		
5	5			25	28			45	110		

6	6			26	30			46	115		
7	7			27	32			47	120		
8	8			28	34			48	125		
9	9			29	36			49	130		
10	10			30	38			50	135		
11	11			31	40			51	140		
12	12			32	45			52	145		
13	13			33	50			53	150		
14	14			34	55			54	155		
15	15			35	60			55	160		
16	16			36	65			56	165		
17	17			37	70			57	170		
18	18			38	75			58	180		
19	19			39	80			59	190		
20	20			40	85			60	200		

- Analysis details of $V_C - t$ plot during discharging.

$$37\%\text{ of } V_s = \text{...} \quad (41)$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{...} \quad (42)$$

$$\tau_{the} = \text{...} \quad (43)$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \% \text{....................} \quad (44)$$

- Analysis details of $I - t$ plot during discharging.

$$I_0 = \text{...} \quad (45)$$

$$37\%\text{ of } I_0 = \text{...} \quad (46)$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{...} \quad (47)$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \% \text{....................} \quad (48)$$

3.3 Part C: $C = 2200\mu F,\ R = 10k\Omega,\ V_s = 10V$

Calculate the following values in detail.

$$\tau = \text{...} \tag{49}$$

$$7\tau = \text{...} \tag{50}$$

Charging

Table 5: Table of time, voltage and current values during charging for $C = 2200\mu F$, $R = 10k\Omega$, $V_s = 10V$.

#	t (*sec.*)	I (...)	V_C (V)	#	t (*sec.*)	I (...)	V_C (V)	#	t (*sec.*)	I (...)	V_C (V)
1	1			21	21			41	90		
2	2			22	22			42	95		
3	3			23	24			43	100		
4	4			24	26			44	105		
5	5			25	28			45	110		
6	6			26	30			46	115		
7	7			27	32			47	120		
8	8			28	34			48	125		
9	9			29	36			49	130		
10	10			30	38			50	135		
11	11			31	40			51	140		
12	12			32	45			52	145		

13	13			33	50			53	150		
14	14			34	55			54	155		
15	15			35	60			55	160		
16	16			36	65			56	165		
17	17			37	70			57	170		
18	18			38	75			58	180		
19	19			39	80			59	190		
20	20			40	85			60	200		

- Analysis details of $V_C - t$ plot during charging.

$$63\% \text{ of } V_s = \text{...} \quad (51)$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{...} \quad (52)$$

$$\tau_{the} = \text{...} \quad (53)$$

$$\text{P.E.} = \%\frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \%\text{.............................} \quad (54)$$

- Analysis details of $I - t$ plot during charging.

$$I_0 = \text{...} \quad (55)$$

$$37\% \text{ of } I_0 = \text{...} \quad (56)$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{...} \quad (57)$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \%\text{....................} \quad (58)$$

Disharging

Table 6: Table of time, voltage and current values during discharging for $C = 2200\mu F$, $R = 10k\Omega$, $V_s = 10V$.

#	t (*sec.*)	I (__)	V_C (V)	#	t (*sec.*)	I (__)	V_C (V)	#	t (*sec.*)	I (__)	V_C (V)
1	1			21	21			41	90		
2	2			22	22			42	95		
3	3			23	24			43	100		
4	4			24	26			44	105		
5	5			25	28			45	110		

6	6			26	30			46	115		
7	7			27	32			47	120		
8	8			28	34			48	125		
9	9			29	36			49	130		
10	10			30	38			50	135		
11	11			31	40			51	140		
12	12			32	45			52	145		
13	13			33	50			53	150		
14	14			34	55			54	155		
15	15			35	60			55	160		
16	16			36	65			56	165		
17	17			37	70			57	170		
18	18			38	75			58	180		
19	19			39	80			59	190		
20	20			40	85			60	200		

- Analysis details of $V_C - t$ plot during discharging.

$$37\% \text{ of } V_s = \text{...} \tag{59}$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{...} \tag{60}$$

$$\tau_{the} = \text{...} \tag{61}$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \% \text{.................................} \tag{62}$$

- Analysis details of $I - t$ plot during discharging.

$$I_0 = \text{...} \tag{63}$$

$$37\% \text{ of } I_0 = \text{...} \tag{64}$$

$$\tau \text{ corresponds to it(i.e. } \tau_{exp}) = \text{...} \tag{65}$$

$$\text{P.E.} = \% \frac{|\tau_{the} - \tau_{exp}|}{\tau_{the}} \times 100 = \% \text{.................................} \tag{66}$$

4 Conclusions

5 Notes

References

[1] Physics Libre Texts. Capacitors and capacitance. [Online; accessed 22-April-2019].

[2] Wikipedia contributors. Capacitor — Wikipedia, the free encyclopedia, 2019. [Online; accessed 22-April-2019].